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Fry plots: warning about summed moments

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Abstract

The sum of first moment of a Fry plot is ordering-invariant, therefore strain-invariant, and so of no use in strain estimation. The sum of second moment appears no better. Therefore summing of moments is not a useful approach to Fry plots.

How best to extract strain information from a Fry plot (Fry, 1979), after you have constructed one? The author has considered many ways to derive parameters which characterise the clustering/anticlustering and the strain. One treatment which fails with notable reliability is the summing of first moments. The reasons are systematic and counterintuitive. They are presented here in the hope of saving others wasted effort.

1. Concepts

Fry plots were developed in the belief that it is better to attain strain information from *all* separations between objects of an object distribution, rather than only from those which lie closest. This is confounded if most points of a Fry plot are disregarded, and the strain ratio and orientation are estimated only from the inner cut-off representing the closest objects. So, the author has searched for features of Fry plots as a whole from which to estimate strain.

In principle, an anticlustered distribution can be created from a purely random distribution by slightly increasing separations of those pairs of objects which lie closest together. These are represented as the points nearest to the centre of a Fry plot, and the effect of increasing object separation is to move them further away, so increasing their contribution to moments. If such adjustments to the distribution are isotropic, so too are the corresponding movements on the Fry plot and their increases in moment. When such an anticlustered distribution is deformed, each of the movements on the Fry plot suffers deformation. Excess moment will no longer be isotropic, and its variation with direction can be used to determine the strain ratio and orientation. At least, that is the idea.

2. An example

Figs. 1 to 3 are derived from the same anticlustered point distribution. Figs. 1(a) and 2(a) are Fry plots, with number of points increasing to the right. Figs. 1(b) and 2(b) represent their first moments, for the complete 2π range of directions in the plane of the paper, scaled relative to an equivalent homogeneous disc.

The left-most plot in Fig. 2(a), where the points lie more than 0.8 radius from its centre, has, about all diameters, moments (Fig. 2b) which are greater than those of an even density disc. In the left-most plot of Fig. 1(a) the points lie anywhere outside only 0.25 of plot radius. The first moments of this plot (Fig. 1b) are in some directions greater, but in others less than those of an even density disc. In both Figs. 1 and 2, these left-most plots show substantial variation of moment with orientation. Had they come from a natural example, it would be tempting to suggest that it had suffered deformation, but that the sample should be enlarged, at two stages in the procedure, to increase confidence in this judgement.

Increasing the area of the object distribution which is sampled will increase the density of points on the

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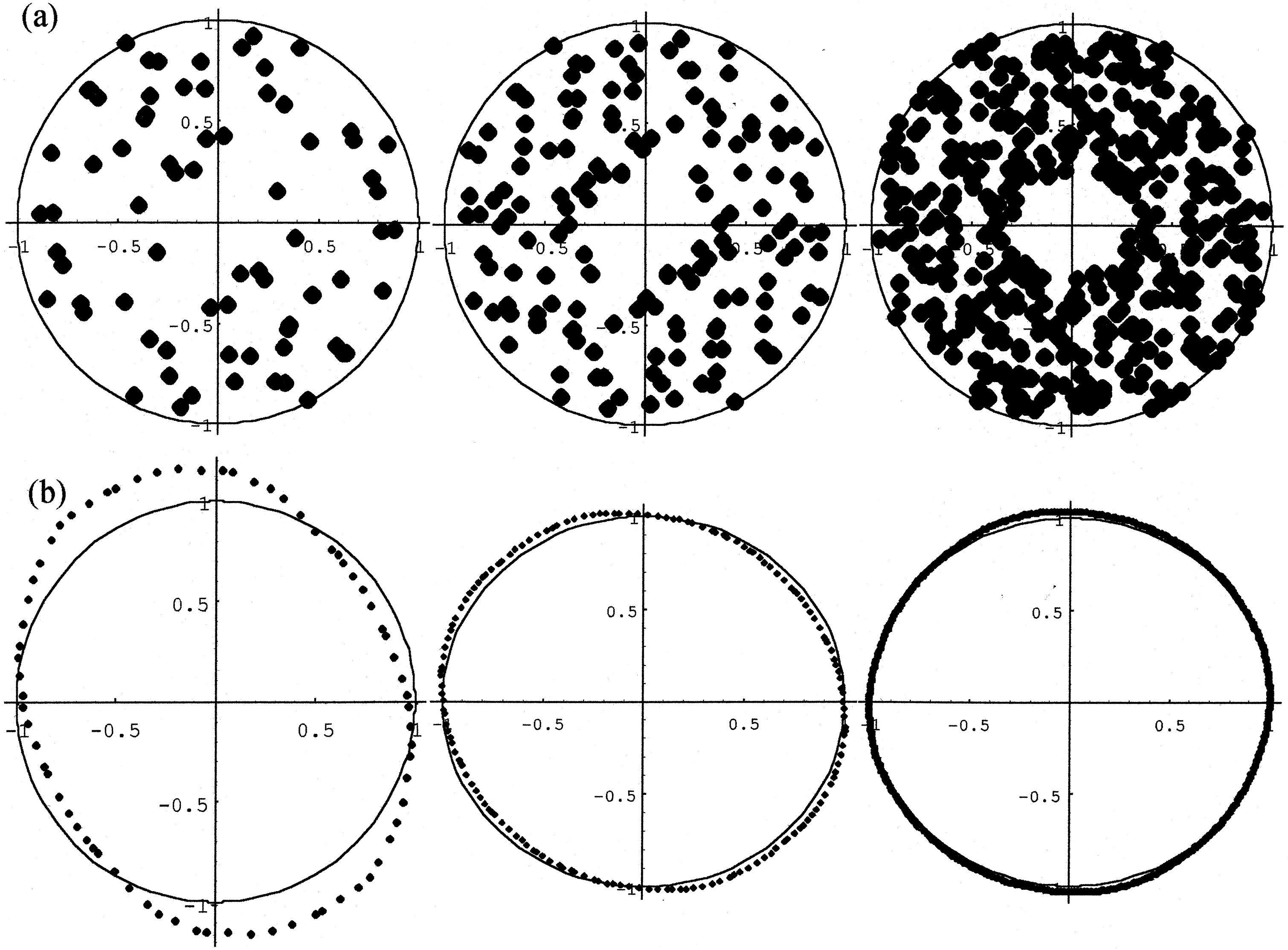

Fig. 1. (a) Fry plots, radius 1.6, from sampling the artificial distribution with absolute cut-off at 0.4 discussed in the text. The area of sample increases from left to right: 36.0; 57.76; 100.0. (b) First moment plots of the Fry plots in (a). All points of the Fry plot are equally weighted. The moment of the half of the Fry plot to one side of any diameter, about that same diameter, is the distance from the same diameter in the first moment plot to the perpendicularly furthest point from it. For example, take the half Fry plot above the horizontal diameter. The sum of first moments of all its points, about this diameter, is given by the height above horizontal diameter of the highest point in the first moment plot. All first moment plots are scaled to a standard circle representing the moment of a homogeneous disc of the same size and average density as the Fry plot.

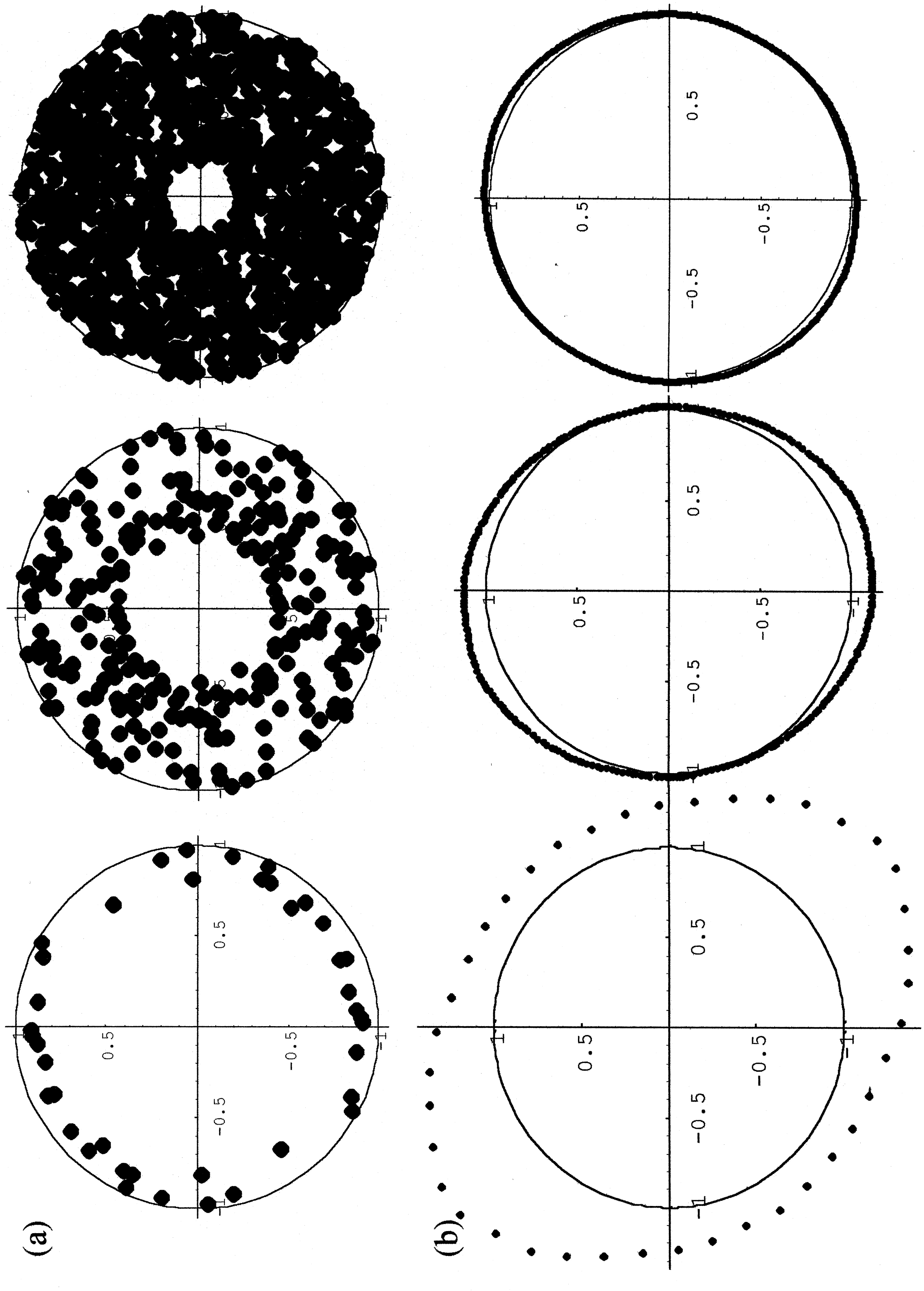

Fig. 2. (a) Fry plots of radius (from left to right) 0.5, 1.0 and 2.0, all from one sample, of area 324, of the distribution, cut-off 0.4, used for Fig. 1. (b) First moment plots of the Fry plots in (a).

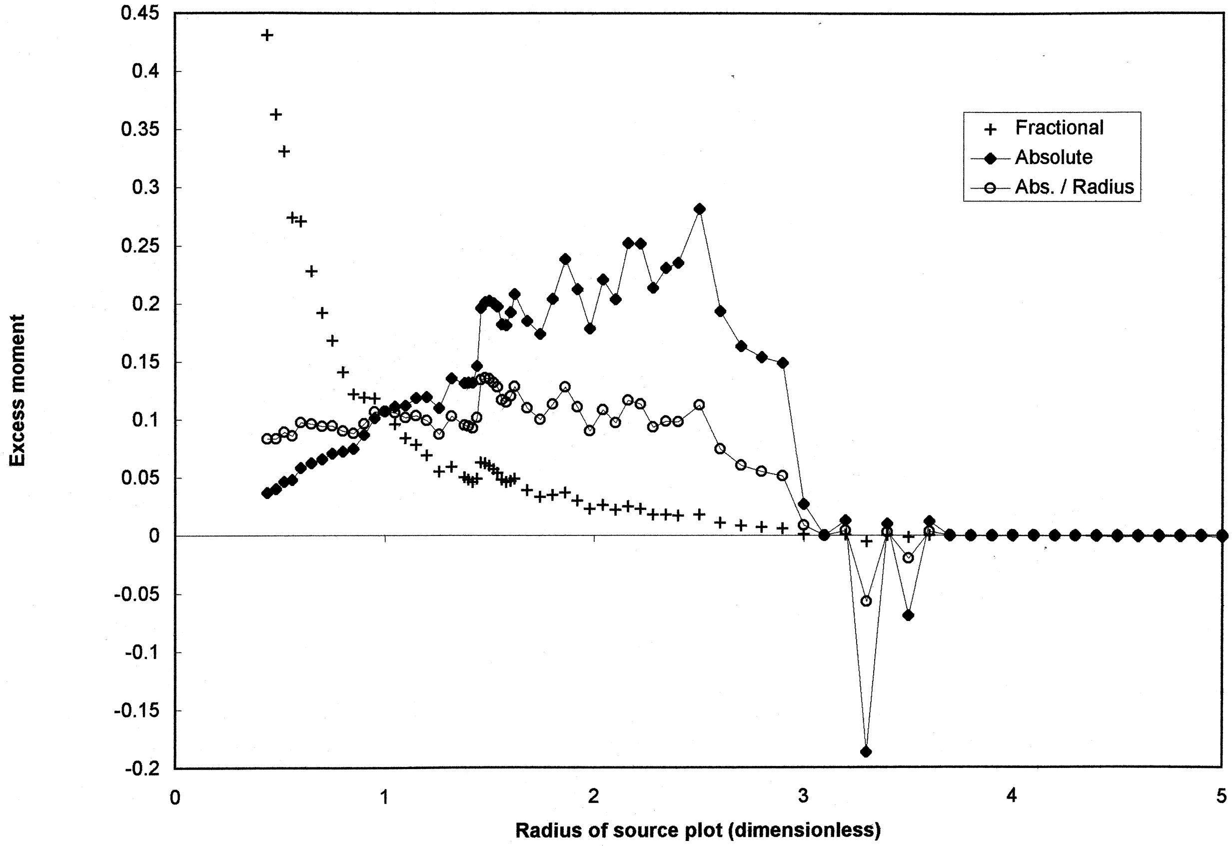

Fig. 3. Excess of first moment of Fry plot, above that of a homogeneous disc of same size and average density, as a function of Fry plot radius. Sample, area 324, of the same distribution as Figs. 1 and 2. Values are approximations to an integration through complete 2π of orientation. 'Fractional' indicates the ratio of the excess to that of the reference disc.

Fry plot. This should suppress correlations between orientation and moment which are merely statistical fluctuations when numbers of points are small. The source population for this example is isotropic, and Fig. 1 shows the relationship between moment and orientation becoming less directional with increasing sample size.

Also, the distance on the sample represented by the radius of the Fry plot should be increased to at least a value, not predictable in advance, beyond which there is no significant coupling of object positions in any direction, and therefore no further increase in excess moment in any direction with increasing radius of plot. Without sufficient radius we cannot have confidence in a strain estimate, because the excess moment may be completely accumulated in one direction (of least stretch) but not in another (of greater stretch). The way in which the first moment plot of any anti-clustered distribution has been found to change qualitatively as the Fry plot radius is increased is illustrated by Fig. 2. The first moment of the sample converges on the reference circle. That is to say, the excess of moment, as a fraction of that of an equally weighted homogeneous disc, tends to zero for all orientations. According to the initial concept, this is not a problem. Though fractional excess moments will decrease as exemplified in Fig. 2, absolute values of excess moment should stabilise and be usable for estimation of strain.

3. The problem

The behaviour of excess moments with increasing radius of Fry plot does not accord with the above expectations. The reason why stabilised values are not illustrated here is that they do not happen. Absolute excess moments do not converge to finite values, either for artificial or natural samples.

The behaviour in the case of the artificial point distribution used for Figs. 1 and 2, is shown in Fig. 3. This distribution was accumulated from a pseudo-random co-ordinate stream by simple filtering. Each added point had to lie further from all previously accepted points than 0.4 dimensionless distance (defined by equating final point density to unity). No other constraint was imposed. Because the distribution is known to be isotropic, its moments can legitimately be integrated through all orientations to reduce the statistical noise in plots, such as Fig. 3, of functions of radius.

The decline in fractional excess moment with radius in Fig. 3 is expected, and the excess moment fluctuations in the range from 0.44 to 1.2 correspond to subjective features of Fry plots. The general form of Fig. 3 for radius above 1.3, though reproducible with other simulations, was unforeseen from theoretical considerations. The reduction in excess moment to near zero corroborates the lack of convergence in studies of natural samples. It implies some error in the underlying concepts, and that first moments can never be used to estimate strain.

4. Explanation

Imagine transforming a point distribution, from complete disorder to being anticlustered, by moving those points too close to their neighbours towards adjacent gaps. Consider first the motion of just one chosen point. The direction it is moved will be taken as the x-axis and the distance moved, δx. Consideration is initially restricted even further, to a pair of thin strips of sample, parallel to x, disposed symmetrically about the chosen point [strips (1) and (2) in Fig. 4a]. Taking strip (1) first, the positions of points in it relative to the chosen point as origin are represented in a Fry plot (Fig. 4b) by an exact copy of this strip. Each point has its Fry plot x-coordinate decreased by δx. Taking now strip (2), for each point in it used successively as origin, the chosen point will be represented in the same strip of the Fry plot as just considered. Together, such points from origins in strip (2) constitute an exact centrosymmetric copy of strip (2) of the sample. Their x-coordinates are all increased by δx.

We make the usual assumptions, that the sample is from a homogeneous isotropic population, that edge effects are avoided, plus a requirement that the length of strip considered substantially exceeds δx (to whatever is the necessary extent). The two expected contributions to the moment of the strip of the Fry plot cancel, and the plot remains centrosymmetric. Generalising now to any direction, at angle ϕ to the x-axis, each point's moment in that direction on the Fry plot is changed $\pm\delta x.\cos\phi$, which, for the combined strip considered, is expected, for any ϕ, to sum to zero over all points positive in this direction from the origin. To summarise, the expected sum of first moments of *any* strip of the Fry plot lying positive from the origin *in any general direction* is conserved. This statement will hold both for the particular case of a strip directly outward from the origin, and for the sum of such strips to form a complete plot.

Because the result is independent of direction of motion of the chosen point, it can be extended indefinitely to the result of as many movements, in any directions, of as many points as are necessary to transform the distribution to any attainable modification, and so generalised to all Fry plots of samples from initially homogeneous isotropic (but not as yet deformed) populations.

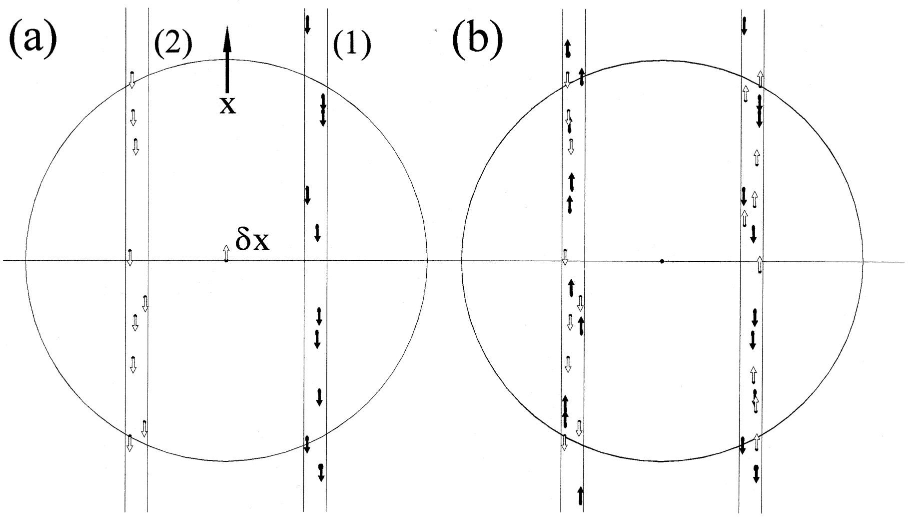

Fig. 4. (a) The parts of a hypothetical sample considered in detail: The view is centred on the point to be moved (by $+\delta x$). The points in the x-parallel strips (1) and (2) have movement relative to the chosen point of $-\delta x$, shown by arrows. (b) The resulting elements of the Fry plot. Points in strip (1) are reproduced in the equivalent strip on the right side of the Fry plot. Motion of the point chosen as centre changes their x-coordinates by $-\delta x$ (dark arrows). The other points in this strip are the position of the central point, relative to each of the points of strip (2). Motion of the central point changes their x-coordinates by $+\delta x$ (light arrows). Symmetrically equivalent operations give a centrosymmetrically equivalent strip on the left of the plot.

Because there is no change in sum of first moments with anticlustering, it is not possible to modify such change by deformation. Therefore, any correlation between summed first moment and direction in a plot from a real sample represents random statistical fluctuation under an unknown transformation. It has no information content from which strain or other parameters may be deduced.

5. Are higher moments an alternative?

Subjectively, one may say that the concentric girdles of higher density around the void at the centre of an anticlustered Fry plot derive not just from points moving out from the centre, but from equal motion inward, away from lower density girdles beyond. On this basis, one would expect, if sum of δx is conserved, that calculated sum of second moments (approximating to $\Sigma x.\delta x$) would change with anticlustering. In practice, the value of this, too, is less than statistical noise. One is led to conclude that the density of points has an oscillation of value away from a Fry plot origin, which is damped rapidly to the level of noise. If so, the condition for the sum of second moment to be conserved is that the amplitude of oscillation (even for a complicated waveform) is proportional to $1/x$. Whether this is merely approximated by anticlustered Fry plots, or is a mathematical requirement, is a question the author has failed to solve. The practical results remain, that moments, of whatever order, do not appear to provide a basis for estimating strain from Fry plots.

References

Fry, N., 1979. Random point distributions and strain measurement in rocks. Tectonophysics 60, 89–105.